THE INFLUENCE OF PYROCLAST SIZE DISTRIBUTIONS ON PYROCLAST ERUPTION SPEEDS IN STEADY EXPLOSIVE ERUPTIONS. Lionel Wilson[1] and Elisabeth A. Parfitt[2] [1]Environmental Sci. Div, Institute of Environmental & Biological Sciences, Lancaster University, Lancaster LA1 4YQ, U.K. L.Wilson@lancaster.ac.uk [2]Earth Sciences Dept., University of Leeds, Leeds LS2 9JT, U.K. L.Parfitt@earth.leeds.ac.uk

We point out a deficiency in earlier mathematical models of steady explosive eruptions (classified as hawaiian or plinian on Earth) which leads to an overestimate of eruption speeds of large clasts for a given exsolved magma gas content - or conversely an underestimate of the magma volatile content needed to produce a given pyroclast dispersion around a vent. The consequences are greatest for eruptions on planets with high atmospheric pressure (Venus and Earth) but are not trivial for any explosive eruptions in the Solar System, even those on differentiated asteroids.

Semi-analytical models of explosive eruptions [1-3] all address the following stages in the ascent of magma: gas bubble nucleation due to supersaturation at depth; magma acceleration due to gas expansion as bubble growth occurs; magma disruption and further expansion of free gas and entrained clasts until *either* atmospheric pressure is reached at the surface *or* choked flow occurs in the vent and a final stage of gas expansion and clast acceleration into the vacuum above the vent occurs. All models use a continuity equation and a momentum equation to track the motion. Most assume an isothermal system, with the heat content of the magmatic liquid buffering the temperature of the expanding gas; other models assume an adiabatic system with poor thermal coupling in the late stages of gas expansion into a vacuum. In all cases, what is calculated for the eruption products is the total energy per unit mass after allowance for the effects of energy sources (expansion of the currently available gas at each depth) and energy sinks (friction at the conduit wall and potential energy to raise the magma against gravity). The residual energy per unit mass, E, is assigned to the erupted gas and pyroclasts as kinetic energy. However, to evaluate correctly the upward speeds through the vent of clasts with a range of sizes it is necessary to distribute the total kinetic energy so that (a) all clasts travel upwards more slowly that the gas phase by an amount equal to their local terminal velocities in the gas, and (b) the total kinetic energy of all the components, including the gas, sums to E. These last steps have been neglected in treatments published so far.

To carry out the required energy partitioning we define a series of N pyroclast size classes covering the clast diameter range. Since pyroclast size distributions are more nearly log-normal than normal, let D_i be the geometric mean diameter of clasts in the ith class, where i runs from 1 to N, and let the median terminal velocity of clasts in this class to be T_i, the terminal velocity of a clast with diameter D_i. T_i is found by equating the gravitational weight of the clast to the drag force exerted on it by the gas, giving $T_i = [(4 D_i \rho g)/(3 C_d \rho_g)]^{1/2}$ when the gas flow around the clast is turbulent, and $T_i = [(D_i^2 \rho g)/(18 \eta_g)]$ when the gas flow around the clast is laminar, where ρ is the clast density, g is the acceleration due to gravity, ρ_g is the gas density (a function of the ambient pressure and temperature), C_d is a friction factor with a value within ~30% of 0.7 and η_g is the viscosity of the gas (mainly a function of temperature). Retrospective evaluation of the Reynolds number of the motion defines which formula is appropriate. The fraction of the total mass of erupted pyroclasts in the ith pyroclast class is F_i, so that $\Sigma F_i = 1$ where 1 i N. A class corresponding to i = 0 is defined for the magmatic gas, which represents a mass fraction M_0 of the total erupted magma. Then if M_i is the fraction of the total mass of erupted material (gas and clasts) in the ith pyroclast class, $1 = M_0 + \Sigma [M_i]$, 1 i N, and we can derive the M_i from the measured F_i using $M_i = (1-M_0) F_i$. If the gas speed in the vent is U_g, the upward speed of clasts in class i is $(U_g - T_i)$ and the total kinetic energy is $\Sigma[M_i (U_g - T_i)^2] + M_0 U_g^2 = 2 E$. Expanding and noting that U_g is a constant, $U_g^2 \Sigma [M_i] - 2 U_g \Sigma [M_i T_i] + \Sigma [M_i T_i^2] + M_0 U_g^2 = 2 E$, or $U_g^2 - 2 U_g \Sigma [M_i T_i] + \Sigma [M_i T_i^2] - 2 E = 0$, which is a quadratic and is soluble analytically for U_g.

Few data exist on total pyroclast grainsize distributions. We use our recent measurements on the Pu'u Puai deposit in Hawai'i to characterise a relatively gas-rich basaltic lava fountain eruption on Earth. The third column of Table 1 gives our measured values of F_i and the fourth column gives the corresponding values of M_i calculated using petrological evidence that the magma exsolved ~0.5 weight % H_2O equivalent , i.e., $M_0 = 0.005$. H_2O has a density of $\rho_g = 0.15$ kg/m^3 at the eruption temperature of ~1400 K. With a typical clast density of $\rho = 1000$ kg/m^3 this gives the terminal velocities in the fifth column of the table, leading to the corresponding products $[M_i T_i]$ and $[M_i T_i^2]$ which sum to $\Sigma [M_i T_i] = 108.2890$ and $\Sigma [M_i T_i^2] = 15759.264$, respectively. To find U_g it is necessary to know E, the kinetic energy per unit mass of eruption products. Based on observations, the mass eruption rate for this event must have been close to 2 x10^5 kg/s. Using this value, M_0 = 0.5 weight %, and a circular conduit geometry, we simulated this eruption using the computer code described in [1] and [2], finding E = 7688 J/kg, so U_g = 214.8 m/s. This must be compared with the result of neglecting the size distribution of the pyroclasts and giving the same eruption speed V to all of the clasts and the gas, such that $E = (1/2) V^2$. Then V = 124 m/s, *nearly a factor of*

2 too small. Using U_g = 214.8 m/s, the eruption speeds U_i through the vent of clasts in each of the size classes in Table 1 were calculated from $U_i = U_g - T_i$ and are listed in the final column of the table. Examination of these values shows that clasts with the largest median size used, 452.5 mm, for which T = 237.3 m/s, cannot in fact reach the level of the vent. This means that the calculation is not quite internally consistent, and that the exsolved magma volatile content must have been a little in excess of 0.5 weight %.

Table 2 shows the results of the equivalent calculations applied to pyroclastic droplets from a lava-fountain eruption on a medium-sized differentiated asteroid such as 4 Vesta [4]. The gas speed found before performing the correction for the clast speeds was 33.9 m/s, within 10% of the corrected value 36.8 m/s, showing that this problem is smallest when pyroclast sizes, and so their terminal velocities, are small, as will be the case when no atmosphere is present. However, this effect will not be trivial at the atmospheric pressure typical of Mars, and needs to be accounted for when relating dispersal of pyroclasts around explosive vents to implied magma volatile contents.

References: [1] Wilson, L., Sparks, R.S.J. & Walker, G.P.L. (1980) GJRAS 63, 117. [2] Wilson, L. & Head, J.W. (1981) JGR 86, 2971. [3] Giberti, G. & Wilson, L. (1990) Bull. Volc. 52, 515. [4] Wilson, L. & Keil, K. (1996) EPSL 140, 191.

Table 1. Basaltic lava fountain on Earth. F_i: fractional mass of pyroclasts in size class with median diameter D_i; M_i: fractional erupted mass which these pyroclasts represent when exsolved magmatic water content is 0.5 weight %; T_i: clast terminal velocity; U_i: clast eruption speed.

clast size range/mm	D_i /mm	F_i	M_i	T_i /(m/s)	M_iT_i /(m/s)	$M_iT_i^2$ /(m/s^2)	U_i /(m/s)
640 - 320	452.5	0.103	0.10248	237.30	24.3185	5570.781	!
320 - 160	226.3	0.206	0.20497	167.80	34.3940	5771.308	47.0
160 - 80	113.1	0.186	0.18507	118.66	21.9604	2605.822	96.1
80 - 40	56.57	0.165	0.16418	83.90	13.7747	1155.698	130.9
40 - 20	28.28	0.124	0.12338	59.32	7.3189	434.157	155.5
20 - 10	14.14	0.082	0.08159	41.95	3.4227	143.582	172.9
10 - 5	7.071	0.062	0.06169	29.66	1.8297	54.270	185.1
5 - 2.5	3.536	0.041	0.04079	20.98	0.8558	17.954	193.8
2.5 - 1.25	1.768	0.021	0.02090	14.83	0.3099	4.597	200.0
1.25 - 0.625	0.8839	0.010	0.00995	10.49	0.1044	1.095	204.3
		1.000	0.99500		108.2890	15759.264	

Table 2. Basaltic lava fountain on the asteroid 4 Vesta. See Table 1 for definitions of variables.

clast size range/µm	D_i /µm	F_i	M_i	T_i /(m/s)	M_iT_i /(m/s)	$M_iT_i^2$ /(m/s^2)	U_i /(m/s)
17 - 55	30	0.2	0.19994	1.0	0.199940	0.1999400	35.8
55 - 173	100	0.2	0.19994	1.0	0.199940	0.1999400	35.8
173 - 547	300	0.2	0.19994	1.1	0.219934	0.2419274	35.7
547 - 1730	1000	0.2	0.19994	1.9	0.379886	0.7217834	34.9
1730 - 5470	3000	0.2	0.19994	10.3	2.059382	21.2116346	26.5
		1.0	0.99970		3.059082	22.5752254	